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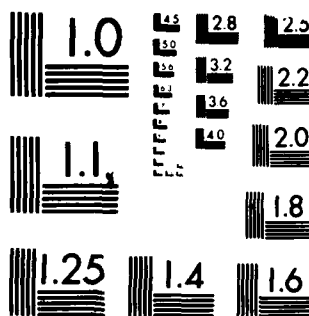
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Numerical Solutions of Non-Uniquely Solvable Integral
Equations Using Finite Rank Modifications

Final Report

by

Sylvia T. Bozeman

May 25, 1984

U.S. Army Research Office
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| 20. ABSTRACT (Continue on reverse side if necessary and identify by block number) Our research has produced a new method for the approximation of eigenfunctions and generalized eigenfunctions of a type of integral operator by using finite rank perturbations: Research continues on the development of a numerical procedure for solving Fredholm integral equations of the second kind that have no unique solution by employing the same type of finite rank modification. | | |

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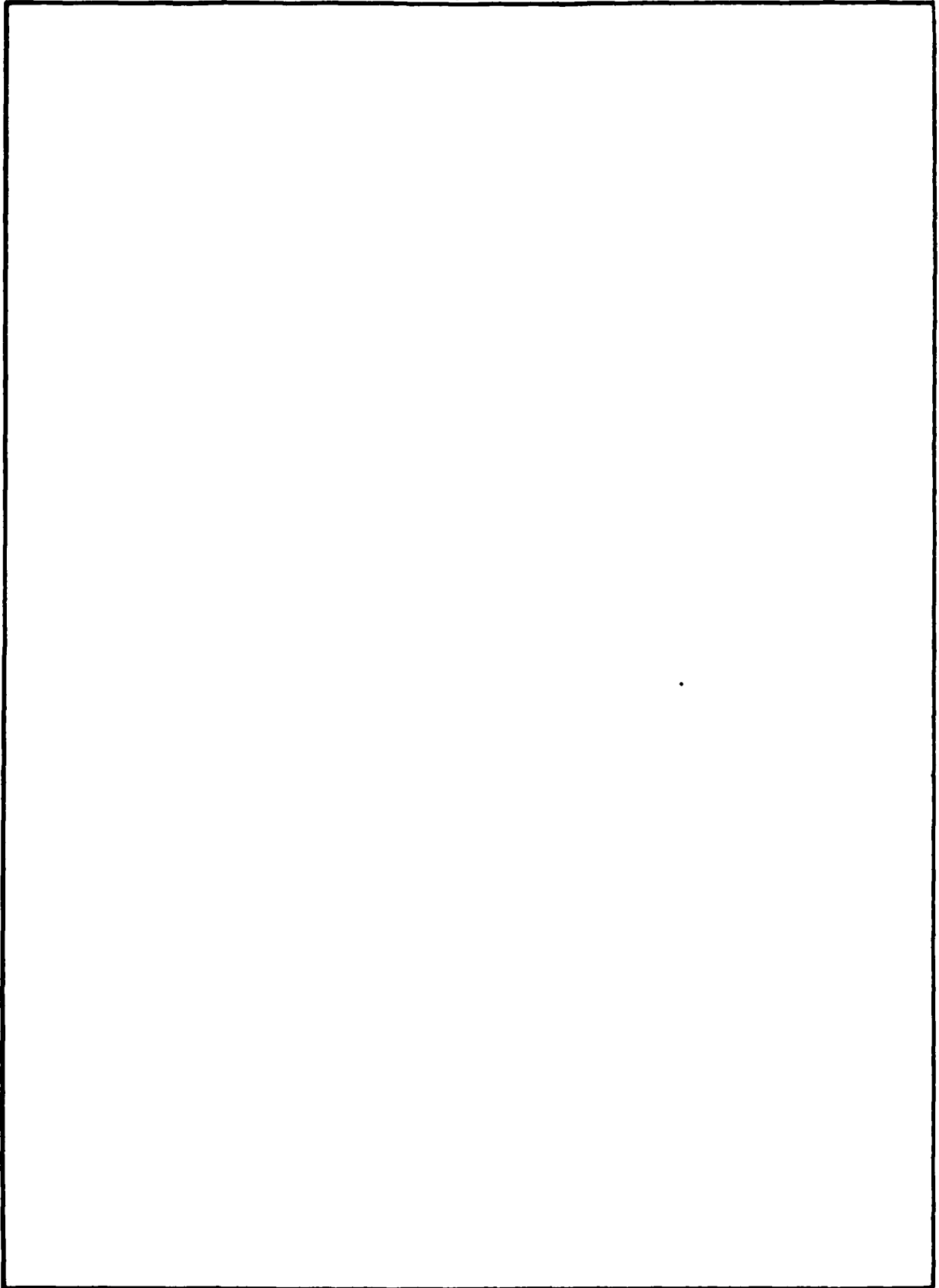
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Many numerical methods have been developed for solving the linear operator equation

$$(I-K)y = f \quad (1)$$

in the case that (1) represents a Fredholm integral equation of the second kind. This research concentrated on the equations of this type when $I-K$ is not an invertible operator. We had two principal objectives. First, we sought to develop a technique for numerically solving (1) by using a finite rank modification to transform the equation into a uniquely solvable one. The usual algorithms and error analysis could then be implemented for the uniquely solvable problem. Our second objective was to produce a new method for the approximation of eigenfunctions of the integral operator K in (1).

A theory of finite rank perturbation of Fredholm operators was developed in Hilbert spaces in [3] and extended to more general Banach spaces in [1]. The techniques developed here are based on that theory. We have been most successful in achieving the second objective. To this end we have used the finite rank perturbations to construct an invertible operator B from the noninvertible operator A where A is a Fredholm operator, e.g., $A \equiv I-K$ in (1). Previous results related the operator B^{-1} to $N(A)$. In this we extended our investigation to the relationship between higher powers of B^{-1} and null spaces of higher powers of A . It is this relationship that allowed us to develop procedures for constructing a basis for the null space $N(A^2)$ and for obtaining independent elements in $N(A)$, $N(A^2), \dots, N(A^r)$, where r is a positive integer. This procedure applied specifically to $A = \lambda I - K$, where K is an integral operator, allowed us to approximate the eigenfunctions and generalized eigenfunctions of K using an approximation $\tilde{\lambda}$ to the eigenvalue λ . These results have been accepted for publication as they appear in [2]. A copy of the manuscript was forwarded to the U.S. Army Research Office at the time that it was submitted to the journal.

Our efforts in achieving the first objective have been expended in developing the appropriate computer software. The technique for solving (1) when $I-K$ is singular has consisted theoretically of constructing a finite rank operator L so that

$$(I-K-L)y = f \quad (2)$$

has a unique solution for all f in some appropriate space. For f in the range of $I-K$ the solution of (2) is then also a solution of (1) (See [1] and [3] for details). The main drawback to the construction L is that the dimension of the null space $N(I-K)$ must be known. If it is not we can try to discover it from the fact that $I-K-L$ is not invertible unless the rank of L is equal to the dimension of $N(I-K)$ and the fact that if f is properly chosen the solution of (2) satisfies certain prescribed conditions [3, Theorem 3.3]. If the dimension of $N(I-K)$ is known then L can in effect be randomly generated and (2) can be solved by conventional methods thus producing a solution of (1). This approach has already proved fruitful in numerically solving equations involving singular matrices (See [4]). In attempting to develop a numerical procedure for determining the dimension of $N(I-K)$ we have concentrated our research on refining the process of numerically determining the dimension of $N(A)$ where A is a singular matrix. Our efforts in this area have not yet produced any significant results. The principal investigator is continuing her research in this area.

Scientific personnel employed on this project

were: Sylvia T. Bozeman, Ph.D., Principal Investigator
Luis Kramarz, Ph.D., Associate Investigator
Joyce Anglin, Graduate Assistant

Joyce Anglin was employed on this project during the period from June 1, 1982 to December 30, 1982. She was awarded the M.S. Degree in Mathematics in August, 1983. Her thesis is entitled "Perturbation Theory and Fredholm Operators".

Since the last progress report the principal investigator has traveled to Louisville, Kentucky to attend the joint national meetings of several mathematical

societies including the American Mathematical Society, the National Association of Mathematicians, and the Mathematical Association of America. The purpose of this trip was to foster exchange with other researchers working in the same area. These meetings were held January 25-28, 1984.

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